

Test 3

MATH 125- Section 7

November 30, 2017

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

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Show all your work!

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1. [15 points] In each case, determine if L'Hopital rule applies then find the limits of parts (a) and (b).

(a)  $\lim_{t \rightarrow 0^+} \left( \frac{1}{t} - \frac{1}{e^t - 1} \right)$

(b)  $\lim_{\ell \rightarrow 1} \frac{e^{1-\ell}}{\cos(1-\ell)}$

(c)  $\lim_{\theta \rightarrow 0} \frac{1 - \cos(\theta)}{\theta^2}$

(d)  $\lim_{x \rightarrow \infty} (1+x)^{1/x}$

2. [25 points] A cone-shaped coffee filter of radius 6 *cm* and depth 10 *cm* contains water, which drips out through a hole at the bottom at a constant rate of 1.5  $\text{cm}^3$  per second. How fast is the water level falling when the depth of the water is 8 *cm*?

3. [20 points] Evaluate the following definite integrals

(a)  $\int_1^2 3x^2 + 4 \, dx$

(b)  $\int_0^3 \sqrt{9 - s^2} \, ds$

4. [20 points] Find the following integrals

(a)  $\int \frac{t^2 + te^t - 1}{t} \, dt$

(b)  $\int \sec \theta (\sec \theta + \tan \theta) \, d\theta$

5. [20 points] Let  $f$  be an odd function, and suppose that  $\int_{-2}^5 f(x) dx = 10$  and  $\int_{-2}^0 f(x) dx = -3$

(a) Find  $\int_2^5 f(x) dx$

(b) Find  $\int_{-2}^2 |f(x)| dx$

6. [20 points] Find the exact positive value of  $c$  if the area between the graph of  $y = c^2 - x^2$  and the  $x$ -axis is 36.

7. [20 points] Let

$$F(x) = \int_0^x te^t dt$$

- (a) Find the intervals where  $F$  is increasing.
- (b) Find the intervals where  $F$  is concave up.

8. [15 points] The graph of the continuous function  $g$  is given in the following figure. Let  $G(t)$  be the antiderivative of  $g(t)$  that satisfies  $G(0) = 5$ . Find all the critical points of  $G$  with their coordinates.

